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RESEARCH PAPER

Limiting stand density and basal area projection models for even-aged *Tecomella undulata* plantations in a hot arid region of India

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Abstract: This paper presents equations for estimating limiting stand density for *T. undulata* plantations grown in hot desert areas of Rajasthan State in India. Five different stand level basal area projection models, belonging to the path invariant algebraic difference form of a non-linear growth function, were also tested and compared. These models can be used to predict future basal area as a function of stand variables like dominant height and stem number per hectare and are necessary for reviewing different silvicultural treatment options. Data from 22 sample plots were used for modelling. An all possible growth intervals data structure was used. Both, qualitative and quantitative criteria were used to compare alternative models. The Akaike's information criteria difference statistic was used to analyze the predictive ability of the models. Results show that the model proposed by Hui and Gadow performed best and hence this model is recommended for use in predicting basal area development in *T. undulata* plantations in the study area. The data used were not from thinned stands, and hence the models may be less accurate when used for predictions when natural mortality is very significant.

Keywords: model evaluation; path invariant algebraic difference form growth function; potential density; qualitative and quantitative criteria; Rajasthan

Introduction

To improve the agricultural productivity and the living conditions of the people in the arid parts of Rajasthan State in India, the *Indira Gandhi Canal* was constructed. Large-scale afforestation activities were planned by the State Forest Department to combat desertification, and plantations of various tree species such as *Tecomella undulata*, *Dalbergia sissoo*, *Eucalyptus camaldulensis* and *Acacia nilotica* were established using the water from the canal. The plantations were of different age groups with varying stand densities.

T. undulata is an important timber tree species found in the Thar Desert of northwest and western India. Distribution of *T. undulata* is limited to the drier parts of the Arabia, southern Pakistan and northwest India up to an elevation of 1 200 m. The species is mainly found to occur in western parts of Rajasthan in India. It is a medium sized tree that produces quality timber and is the main source of timber amongst the indigenous tree species of hot desert regions. Its wood is strong, tough and durable, which takes a fine finish. The wood is excellent for firewood and charcoal (Anon 1994).

T. undulata is a deciduous or nearly evergreen tree of arid and semi-arid regions. It occurs on flat and undulating areas including gentle hill slopes and sometimes also in ravines. It is well adapted to drained loamy to sandy loam soil having pH 6.5–8.0. The species thrives very well on stabilized sand dunes, which experience extreme low and high temperatures (Anon 1994). It grows in areas of scanty rainfall (annual 150–500 mm) and high temperature (35°C to 48°C). It can withstand extreme low temperature (0°C to −2°C) during winter and high temperature (48°C to 50°C) in summers. The tree is a strong light demander. It is drought and frost resistant as well as fire and wind hardy. At the time of flowering (December-February) it produces beautiful showy flowers in yellow, orange and red colours. *T. undulata* is an accepted tree species in agro-forestry and large population is found in agricultural lands. Information about the growth and yield aspects of *T. undulata* is not available and hence development of growth & yield models for this particular species has become very important.

Populations of trees growing at high densities are subject to density-dependent mortality or self-thinning (Yoda et al. 1963; Westoby 1984). For a given average tree size, there is a limit to the number of trees per hectare that may co-exist in an even-aged stand. The relationship between the average tree size and the number of live trees per unit area may be described by means of

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a limiting line. Estimating the potential density of forest stands, in terms of the surviving trees per hectare, is a central element of growth modelling. It is also one of the most difficult problems to solve, mainly because suitable data from untreated, densely stocked stands are rarely found.

The stand basal area is an important density measure, which simultaneously takes into account the average tree size and the number of trees per unit area. Basal area is being used to analyse the relationship between stand density and tree growth (Assmann 1970). Moreover, in combination with the number of trees, basal area can be used to define the type and weight of a thinning (Gadow and Hui 1999; Staupendahl 1999). Models for stand basal area development have been generated by various workers using a differential equation or the path invariant algebraic difference form of a non-linear equation (Schumacher 1939; Pienaar and Shiver 1986; Souter 1986; Hui and Gadow 1993; García 1994; Rodriguez Soalleiro 1995; Kvist Johannsen 1999).

Growth modelling is a crucial pre-requirement to evaluate the consequences of a particular management action on the future development of a forest ecosystem. Appropriate growth and yield models are not available for many indigenous tree species in India. The aim of this paper is to develop limiting density equation and path invariant algebraic difference form basal area prediction model for even-aged stands of *T. undulata* grown in the hot arid region of Rajasthan State of India.

Materials and methods

Study area

The study was conducted on *T. undulata* plantations in the Indira Gandhi Canal Project area of Rajasthan State in India, which is a part of the Great Indian Thar Desert. The study area is characterized by a large variation in the diurnal and seasonal temperatures. Summer temperature often exceeds 46°C–48°C, especially during May-June. The mean monthly temperature in the area varies between 39.5°C and 42.5°C, while the mean monthly minimum temperature varies between 14°C and 16°C. The soil temperature often reaches 62°C during May and June and often exceeds the air temperature by at least 10°C. The mean annual rainfall varies between 150 to 300 mm. The mean monthly relative humidity fluctuates greatly during the year between 15% and 80%. The mean evaporation varies from 2.7 mm to 4.7 mm per day in winter and from 13.2 mm to 15.3 mm per day in summer. Wind speeds as high as 130 km per hour may be experienced during the summer months. The terrain of the area is very undulating and is frequently infested with the moving sand dunes. A semi-consolidated lime concretionary or gypsum strata is found underneath at many places. The soil is rich in potash but poor in nitrogen and low in organic matter. The soils are coarsely textured and the water retention capacity is low.

Data and field procedure

Twenty-two sample plots (of the size of approximately 0.1 ha) were laid out in the study area covering the available range of the tree ages and stand densities. The plantation densities varied from 450 to 2 188 trees·ha^{-1} while the age variation was from 14 to 21 years. The study was initiated in August 2005 and annual measurement was prescribed in the plots. Thus, the data were available for three annual measurements. The plot data included a record of the age (A), the dominant stand height (H), the quadratic mean diameter (D_g), the stems/ha (N), the basal area/ha (BA). The summary statistics of the pooled data of all the plots are given in Table 1.

Table 1. Summary statistics for the pooled data of the 22 plots of *T. undulata*

Attribute	Age (years)	Dominant stand height (m)	Stems per hectare	Quadratic mean diameter (cm)	Basal area (m^2/ha)
Minimum	14	4.43	450	6.12	1.84
Maximum	21	8.64	2188	12.32	`4.31
Mean	17.79	6.09	1167	8.39	6.7
Standard deviation	1.55	1.17	422.47	1.57	3.48

Figure 1 presents the development of the dominant height, basal area, and stems per hectare over the stand age. It may be inferred from Fig. 1(a) that the data for the plots are concentrated mainly in the ages from 14 to 21 years. Fig. 1(b) shows a decrease in basal area of some of the plots. A considerable decline in stems/ha has been observed in some stands as visible from Fig. 1(c). There was no thinning done in the stands and the decease observed in stem numbers and basal area is due to removal of trees in some plots by local rural people for their own use and mortality due to entomo-pathogenic problems and over-crowding at few places.

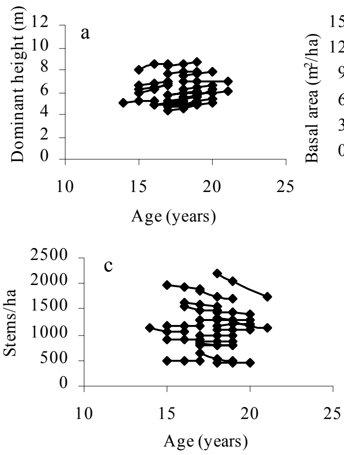

Fig. 1 Graphic representation of the data; (a) height development, (b) basal area development, and (c) stem number development

Limiting density

The relationship among the quadratic mean diameter (D_g), dominant height (H) and the number of stems per unit area (N) may be presented as follows (Goulding 1972):

$$D_g = \frac{1}{\alpha_0 H^{\alpha_1} N + \beta_0 H^{\beta_1}} \quad (1)$$

where, α_0, α_1, β_0, β_1 are parameters that are to be estimated.

The per-hectare basal area is:

$$BA = \frac{\pi}{4} D_g^2 * N = \frac{\pi * N}{4\left[\alpha_0 H^{\alpha_1} N + \beta_0 H^{\beta_1}\right]^2} \quad (2)$$

The stems per hectare at maximum basal area may be obtained by setting the first derivative of equation 2 with respect to N equal to zero (Sterba 1975):

$$N_{G\max} = \frac{\beta_0}{\alpha_0} H^{(\beta_1 - \alpha_1)} \quad (3)$$

Substituting N in equation 1, one can obtain the quadratic mean diameter at maximum basal area:

$$D_{g_{G\max}} = \frac{1}{2\beta_0 H^{\beta_1}} \quad (4)$$

Solving the equation 4 for H and substituting the expression in equation 3 we finally obtain the stems per hectare at maximum basal area (Sterba 1987):

$$N_{G\max} = \frac{\beta_0}{\alpha_0} (2\beta_0)^{\frac{\alpha_1}{\beta_1} - 1} D_{g_{G\max}}^{\frac{\alpha_1}{\beta_1} - 1} \quad (5)$$

The resulting equation 5 represents the limiting relationship. Equation 5 may be simplified as $N_{G\max} = a * D_{g_{G\max}}^{-b}$ and, thus, this equation is similar to the -3/2 power law of self-thinning theory.

Basal area models

In modelling basal area, the path invariant algebraic difference form of growth functions has been applied. Five such equations were selected from literature based on their wider applicability.

Pienaar and Shiver (1986) developed a growth function, which makes it possible to forecast basal area at a given age as a function of previous basal area, age, height and stem number:

$$\ln(BA_2) = \ln(BA_1) + \alpha * \left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \beta * (\ln N_2 - \ln N_1) + \gamma * (\ln H_2 - \ln H_1) + \delta * \left(\frac{\ln H_2}{A_2} - \frac{\ln H_1}{A_1}\right) \quad (6)$$

where BA_1 and BA_2= basal area at age A_1 and A_2; H_1 and H_2= top height at age A_1 and A_2; N_1 and N_2= number of stems at age A_1 and A_2; α, β, γ and δ= model parameters

Forss et al. (1996) used following modified version of equation 6 for modelling basal area growth in *Acacia mangium* plantations in Indonesia:

$$\ln(BA_2) = \ln(BA_1) + \alpha * \left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \beta * (\ln N_2 - \ln N_1) + \gamma * (\ln H_2 - \ln H_1) \quad (7)$$

Hui and Gadow (1993) developed the following equation for projecting a known basal area for stands of *Cunninghamia lanceolata* of varying density:

$$BA_2 = BA_1 * N_2^{1 - \alpha * H_2^{\beta}} * N_1^{\alpha * H_1^{\beta} - 1} * \left(\frac{H_2}{H_1}\right)^{\gamma} \quad (8)$$

Schumacher (1939) proposed following age dependent basal area model that was later used by Schumacher and Coile (1960), Clutter (1963), and Sullivan and Clutter (1972):

$$\ln(BA_2) = \alpha + (\ln BA_1 - \alpha) * \left(\frac{A_1}{A_2}\right) \quad (9)$$

Souter (1986) presented the following model based on Schumacher's equation:

$$\ln(BA_2) = \left(\frac{A_1}{A_2}\right) * \ln BA_1 + \alpha * \left(1 - \frac{A_1}{A_2}\right) + \beta * \left(\ln N_2 - \left(\frac{A_1}{A_2}\right) * \ln N_1\right) \quad (10)$$

To fit the algebraic difference equations, different data structures can be used (Borders et al. 1988; Cao 1993). The accuracy and precision of the predictions depend on both the equation and the data structure but, in general, the method involving all possible growth intervals provided most stable and consistent results (Goelz and Burk 1992). All possible growth intervals data structure uses all possible differences in both directions. In the present study also, all possible intervals have been taken, however, differences were taken only in forward direction. The model parameters were estimated with the help of the STATISTICA statistical package using the Gauss-Newton method.

When using the all possible growth intervals data structure, standard errors for the parameter estimates will be too small and underestimated because the number of observations is artificially inflated. The standard errors should be expanded by √[n(apd)/n(fd)], where n(apd) is the number of observations using all possible differences and n(fd) is the number of observations if using only first differences (Goelz and Burk 1996).

Model evaluation

The comparison of the five basal area models fitted was based on graphic and quantitative analysis of the residuals. Graphical analysis of residuals searching for discrepancies or patterns is an important step in evaluating the fitted models (Gadow and Hui 1999). Residuals were graphically examined to check for any trend. Linear regression of predicted on observed values was also done to see the performance of the fitted models. The ideal value for the intercept and slope of the linear regression is 0 and 1, respectively. A useful test for bias is the simultaneous F-test for slope and zero intercept (Vanclay 1994). The test is based on a

linear regression of observations (y_i) against predictions ($\hat{y}_i$) of the form $y = a + b \cdot \hat{y}$. For testing the hypothesis that α=0 and β=1, the test criterion F is calculated as follows:

$$F_{2,n} = \frac{(n-2)\left[\left(a\sum y_i + b\sum \hat{y}_i y_i\right) - \left(2\cdot\sum \hat{y}_i y_i - \sum \hat{y}_i^{\,2}\right)\right]}{2\cdot\left[\sum y_i^{\,2} - \left(a\sum y_i + b\sum \hat{y}_i y_i\right)\right]}$$

where n = number of sampling units. F_α is the threshold value of F (normally at α=5%). The hypothesis is rejected, when $F_{2,n}>F_\alpha$. The quantitative evaluation of models is also a very important part of growth modelling. The mean residual (MRES), a measure of average model bias, describes the directional magnitude, i.e. the size of expected under- or overestimates. The root mean square error (RMSE) and the variance ratio (VR) are the indices of model precision. The root mean square error is based on the residual sum of squares, which gives more weight to the larger discrepancies. The variance ratio measures the estimated variance as a proportion of the observed one. The other criteria used in model evaluation are the adjusted coefficient of determination (R^2_{adj}), the model precision (MPR) and Akaike's information criterion differences (AICd). The adjusted coefficient of determination shows the proportion of the total variance that is explained by the model, adjusted for the number of model parameters and the number of observations. The model precision (MPR) is a standardised sum of squares criterion proposed by Freese (1960) for evaluating the precision of the fitted model. Akaike's information criterion differences (AICd) is an index for selecting the best model based on minimizing the Kullback-Liebler distance (Burnham and Anderson 1998).

Mean residual (ideal value 0):

$$MRES = \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)}{n}$$

Root mean squared error (ideal value 0):

$$\text{RMSE}= \sqrt{\sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{n - p}}$$

Variance ratio (ideal value 1):

$$VR = \frac{\sum \left(\hat{y}_i - \overline{\hat{y}}_i\right)^2}{\sum \left(y_i - \overline{y}\right)^2}$$

Adjusted coefficient of determination:

$$R^2{}_{\text{adj}} = 1 - \frac{n-1}{n-p}\sum_{i=1}^{n} \frac{\left(y_i - \hat{y}_i\right)^2}{\left(y_i - \overline{y}_i\right)^2}$$

Model precision (ideal value 0):

$$MPR = \frac{1}{\sigma^2}\sum (y_i - \hat{y}_i)^2$$

Akaike's information criterion differences:

$$\text{AICd} = n\ln\hat{\sigma}^2 + 2l - \min\left(n\ln\hat{\sigma}^2 + 2l\right)$$

where, y_i , $\hat{y}_i$ and $\overline{y}_i$ are the measured, predicted and average values of the dependent variable, respectively, n the total number of observations used to fit the models, σ^2 the variance of y; p the number of model parameters; $l = p+1$, and $\hat{\sigma}^2$ the estimator of the error variance of the model, the value of which is obtained as follows:

$$\hat{\sigma}^2 = \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{n}$$

Results and discussions

Potential density

The data collected from the annual measurements in the 22 plots of *T. undulata* were used to fit equation 1 to develop the relationship between the quadratic mean diameter, dominant height, and number of stems per hectare. The estimated parameters and related statistics are given in Table 2. These values of the coefficients were further used to obtain limiting line of maximum basal area through equation 5. The relationship between the quadratic mean diameter and the number of living trees per unit area along with the limiting line is shown in Fig. 2. The solid line represents the potential density, i.e., the maximum stems/ha of the plots with respect to the quadratic mean diameter at maximum basal area. In practice the limiting relationship is difficult to determine, because some trees may die although the limiting density has not been reached (Gadow and Bredenkamp 1992).

Table 2. Parameter estimates and related statistics obtained by fitting equation 1 on the data set

Coefficients	Values	R^2	RMSE
α_0	0.000006		
α_1	1.640970	0.703	0.86309
β_0	1.096503		
β_1	-1.364750		

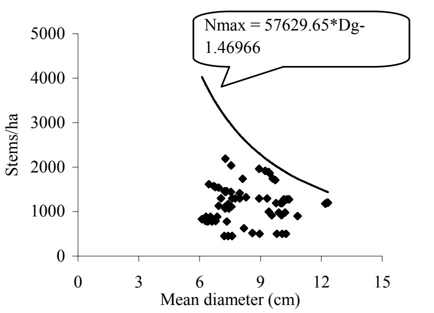

Fig. 2 Relationship between stems/ha and quadratic mean diameter (the solid line is limiting density line and the equation is derived from equation 5 using parameter values given in Table 2)

Basal area models

Equations (6–10) were applied on the data set containing all possible interval data, and model coefficients were estimated

using the Quasi-Newton method of non-linear regression techniques. The parameter estimates and fit statistics are shown in Table 3. The standard errors of the coefficients given in Table 3 indicate that all model coefficients are significant at the 5% level. However, parameter ‘α’ of equations 6, 7 and 10 and parameter ‘γ‘ of equations 6 and 8 became insignificant at 5% level (Table 3) when the standard errors are expanded as proposed by Goelz and Burk (1996). The fit statistics shows that all the models produced high values for the coefficients of determination, adjusted for the number of model parameters and number of observations. Model 8 yielded low values for the root mean squared errors (RMSE) relative to the other models tested. In general, model 8 resulted in high value of R^2_{adj} and low value of RMSE compared to others. Although this equation performed best, the superiority of any model cannot be established only on the basis of these fit statistics. Therefore all the models were subjected to a quantitative evaluation based on the other statistical criteria described under ‘Model evaluation’ of “Materials and Methods’ section. The values of these criteria are presented in Table 4.

Table 3. Parameter estimates and fit statistics for the basal area projection models (equations 6-10)

Equations	α	β	γ	δ	Adj. R^2	RMSE
6	20.89020*	0.44446	1.19753*	11.71727	0.9946	0.25638
	(9.94110)	(0.08721)	(0.40830)	(5.36295)		
7	3.01655*	0.43478	2.02268		0.9943	0.26392
	(1.27448)	(0.08964)	(0.17099)			
8	0.19396	0.56317	3.94394*		0.9949	0.25106
	(0.05904)	(0.25562)	(1.26311)			
9	2.80490				0.9834	0.45148
	(0.09195)					
10	-1.05793*	0.56314			0.9863	0.40993
	(0.52036)	(0.14674)				

Parameters became insignificant at the 5% level when the expansion term proposed by Goelz and Burk (1996) was used. Figures in parentheses are the standard errors of the parameter estimates.

Table 4. The estimated values for the statistical criteria considered to test and compare the five basal area projection models

Models (equations)	MRES	VR	MPR	AICd
6	-0.00557	0.99337	0.00535	3.69284
7	0.01134	1.01499	0.00567	6.59022
8	-0.01237	0.99670	0.00514	0
9	-0.03600	0.95076	0.01660	75.55683
10	-0.00450	0.98518	0.01369	63.77253

The values in Table 4 show that equation 10 has minimum bias (MRES) followed by equation 6. All other models have slightly larger bias. All equations, except equation 7, overestimated the prediction. The models are statistically sound in prediction if they give the values for MPR and VR close to 0 and 1, respectively. It can be seen that equation 8 is meeting these conditions better, closely followed by equation 6. Other equations show greater deviations from the ideal values. Equation 9 produced the largest values for all the criteria and hence is the poorest performer among all the equations used. The values of Akaike’s criterion differences (AICd) also suggest that equation 8 is the best approximating model. Overall, the statistical criteria used for model evaluation clearly reflected the superiority of the Hui and Gadow model (equation 8).

As a rule of thumb, models for which AICd ≤ 2 have substantial support and should receive consideration in making inferences. Models having AICd of about 4 to 7 have considerably less support, while models with AICd > 10 have either essentially no support, and might be omitted from further consideration, or at least those models fail to explain some substantial explainable variation in the data (Burnham and Anderson 1998).

The values of AICd (Table 4) suggest that, in addition to equation 8, equation 6 also has some support (AICd=4.25) for consideration while making inferences. The values of AICd also suggest that the equations 9 and 10 failed badly in explaining some substantial explainable variation in the data and hence should not be considered during model selection procedure.

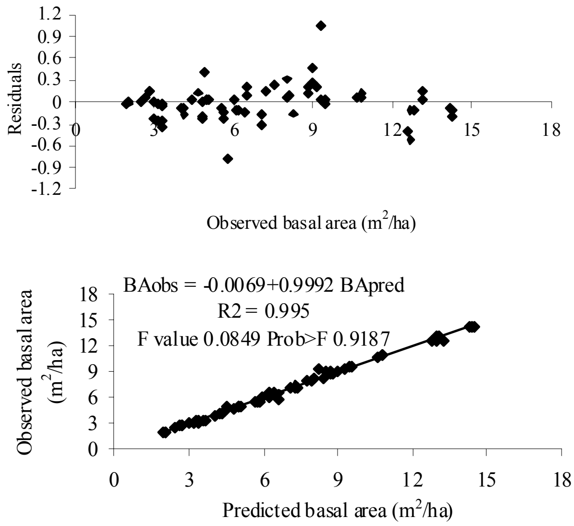

Fig. 3 (a) Plot of residuals over predicted values; (b) plot of observed vs. predicted values for basal area projection model (equation 8)

An important step in evaluating the fitted equations is to perform a graphical analysis of residuals and to identify dependencies or patterns, which indicate systematic discrepancies. The residuals obtained by fitting equation 8 were plotted over observed basal area values. Plots of observed versus predicted values were also generated. These are shown in Fig. 3(a) and (b). Figure 3(a) shows that residuals were randomly distributed and the range of residuals was from -0.79 to 1.05 while Fig. 3(b) shows excellent fit of the model over the entire range of the observed data. A linear model was fitted and the F-value and associated probabilities for the simultaneous test for intercept = 0 and slope = 1 were calculated. The results of the simultaneous F-test (Fig. 1b) show that there are no reasons to reject the null-hypothesis of intercept = 0 and slope = 1 meaning that there are no systematic over or underestimates in the model.

Conclusions

Equation 1 was used to fit the data collected from the 22 sample

plots in pure even-aged stands *T. undulata*, laid out in the hot arid region of Rajasthan State in India, for developing the relationship between mean tree diameters and surviving stems per hectare. The estimated coefficients, in turn, were used to construct the limiting line, indicating the maximum number of stems expected in the stand with respect to the quadratic mean diameter at maximum basal area. This relationship will help in generating information about the number of trees per hectare that should remain in the stands given the mean diameter of the trees in the stands.

Five different equations were tried to model basal area projection in the stands, and Hui and Gadow model (equation 8) performed better compared to others based on selected quantitative statistical criteria. In combination with the stand density, basal area projection model may be used to define the type and weight of thinnings in the stands more appropriately. Thus the models presented in the paper are very crucial in evaluating silvicultural treatment options.

There are also some limitations to the models presented in this paper. The data used were not from thinned stands, and observed decrease in the trees in the plots was due to natural mortality, removal of trees by the villagers or insect-pest and disease problems. The models may be less accurate when used for predictions when natural mortality is very significant (high density plantations and for long projection intervals). Moreover, the data available were for a limited age group between 14 to 21 years of age and hence density is confounded with age. If the models proposed are applied outside the range of density and age of the dataset used in the study, it may produce inaccurate predictions and hence may require some adjustment. Thus, the models may be applied outside the data range, though with caution. If there is no or low mortality, which may be a consequence of heavy or closely spaced thinning events, we can approximate that there will be no change in number of stems ha^{-1} at age A_1 and age A_2 and $N_1 = N_2$. In this case basal area equations used can further be simplified.

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